PrashantEdu

Grades 7–8

Sample Math Practice

with Solutions

QUESTIONS

Easy Questions (1–7)

1. Three boxes contain apples, oranges, and mixed fruit respectively, but ALL labels are wrong. You may draw one fruit from one box only. From which box should you draw to correctly label all three boxes?

(A) The box labeled "Apples" (B) The box labeled "Oranges"

(C) The box labeled "Mixed" (D) Any box will work (E) Impossible

2. Among four friends, exactly one always tells the truth. Ali says: "I am the truth-teller." Ben says: "Ali is lying." Cal says: "Ben is lying." Dan says: "Cal is the truth-teller." Who is the truth-teller?

(A) Ali (B) Ben (C) Cal (D) Dan (E) Cannot determine

3. Ten coins lie on a table, all showing heads. Each move, you must flip exactly three coins. What is the minimum number of moves needed so all coins show tails?

(A) 3 (B) 4 (C) 5 (D) 6 (E) It is impossible

4. Four cards lie on a table showing: A, K, 4, 7. Each card has a letter on one side and a number on the other. Someone claims: "If a card has a vowel on one side, it has an even number on the other." Which cards must you turn over to test this claim?

(A) A only (B) A and 4 (C) A and 7 (D) A, K, and 4 (E) All four

5. A rope fits tightly around the Earth's equator (about 40,000 km). If we add just 10 meters to the rope and lift it uniformly all around, how high off the ground will it be?

(A) Less than 1 mm (B) About 1 cm (C) About 1.6 m (D) About 16 m (E) About 160 m

6. At a party, seven people each shake hands with some others. Is it possible that each person shakes hands with an odd number of people?

(A) Yes, if exactly 3 shake hands (B) Yes, if exactly 5 shake hands

(C) Yes, always possible (D) No, impossible (E) Depends on the arrangement

7. What is 142857×7?

(A) 999,999 (B) 1,000,000 (C) 1,000,001 (D) 999,993 (E) 1,142,857

Medium Questions (8–14)

8. Two players take turns placing identical coins on a circular table. Coins cannot overlap. The player who places the last coin (after which no more can fit) wins. Does the first or second player have a winning strategy?

(A) First player wins (B) Second player wins (C) Depends on table size

(D) Depends on coin size (E) Neither has a guaranteed strategy

9. A chocolate bar consists of 48 small squares arranged in a rectangle. You can break it along any straight line between squares. What is the minimum number of breaks needed to separate all 48 squares?

(A) 6 (B) 12 (C) 24 (D) 47 (E) 48

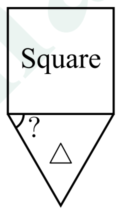
10. Using only 5-cent and 7-cent stamps, what is the largest postage amount that cannot be made?

(A) 19 cents (B) 23 cents (C) 27 cents (D) 29 cents (E) 35 cents

11. Simplify $\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\cdots}}}}$ where the pattern continues infinitely.

(A) 2 (B) $\sqrt{6}$ (C) 3 (D) $\frac{1+\sqrt{25}}{2}$ (E) $\sqrt{7}$

12. An equilateral triangle and a square share a common side. What is the measure of the angle formed between a non-shared side of the triangle and an adjacent side of the square?



(A) 120° (B) 135° (C) 140° (D) 150° (E) 160°

13. Two diagonally opposite corners are removed from an 8×8 chessboard, leaving 62 squares. Can this board be covered exactly by 31 dominoes, where each domino covers two adjacent squares?

(A) Yes, start from a corner (B) Yes, use an L-shaped pattern

(C) No, parity prevents it (D) No, geometry prevents it (E) Depends on domino orientation

14. An investment gains 10% one year and loses 10% the next. After these two years, is the investment worth more, less, or the same as at the start?

(A) Worth more (B) Worth less (C) Worth exactly the same

(D) Depends on the order (E) Depends on the initial amount

Hard Questions (15–21)

15. A calculator shows 10^{100}, then subtracts 47. If you add up all the digits in the display, what do you get?

(A) 883 (B) 890 (C) 891 (D) 892 (E) 900

16. Trains leave stations A and B (500 km apart) toward each other at 50 km/h and 100 km/h. A bird starts from A with the faster train and flies at 150 km/h between the trains until they meet. How far does the bird fly?

(A) 400 km (B) 450 km (C) 500 km (D) 550 km (E) 600 km

17. A census taker asks a woman how old her three children are. She says: "The product of their ages is 36, and the sum equals our house number." The census taker looks at the house number and says: "I need more information." She replies: "My oldest plays piano." What are the children's ages?

(A) 1, 6, 6 (B) 2, 2, 9 (C) 2, 3, 6 (D) 1, 4, 9 (E) 3, 3, 4

18. You have a balance scale and weights of 1, 3, 9, and 27 grams. You can place weights on either side of the scale. What is the heaviest object (in whole grams) you can weigh exactly?

(A) 27 (B) 30 (C) 36 (D) 40 (E) 81

19. Five students each put their name in a hat. Each student then draws a name at random. In how many ways can this be done so that no student draws their own name?

(A) 24 (B) 36 (C) 44 (D) 48 (E) 53

20. A lily pad doubles in size each day. On day 30, it covers the entire pond. On which day did it cover exactly one-eighth of the pond?

(A) Day 3 (B) Day 4 (C) Day 10 (D) Day 27 (E) Day 28

21. Jar A contains 100 ml of milk. Jar B contains 100 ml of water. You transfer one spoonful from A to B, stir thoroughly, then transfer one spoonful from B back to A. Which is greater: the amount of milk now in B, or the amount of water now in A?

(A) More milk in B (B) More water in A (C) They are equal

(D) Depends on spoon size (E) Cannot determine

SOLUTIONS

Easy Questions (1–7)

1. This is a classic logic puzzle. The key insight: since ALL labels are wrong, each box contains something different from its label.

Draw from the "Mixed" box.

Why? Since the label is wrong, this box contains ONLY apples or ONLY oranges (not mixed).

Say you draw an apple. Then:

- The "Mixed" box contains only apples
- The "Apples" box (wrong label) contains either oranges or mixed. But we know apples are in the "Mixed" box, so "Apples" cannot contain mixed (only one box has each). So "Apples" contains oranges.
- The "Oranges" box must contain mixed (the only option left)

One draw from "Mixed" determines everything!

Drawing from "Apples" or "Oranges" would not work because if you draw, say, an orange from "Apples," you still would not know if it contains only oranges or mixed.

Answer: (C) The box labeled "Mixed"

2. Exactly one person is the truth-teller (knight).

If Ali is the knight: He truthfully says he is the truth-teller. But then Ben (who says Ali is lying) is lying, Cal (who says Ben is lying) is telling truth, and Dan (who says Cal is truth-teller) is telling truth. That gives 3 truth-tellers. Contradiction.

If Ben is the knight: Ben says Ali is lying (true, since Ali falsely claims to be the truth-teller). Cal says Ben is lying (false, so Cal is a liar). Dan says Cal is truth-teller (false, so Dan is a liar). This works: only Ben tells truth.

Ben is the truth-teller.

Answer: (B) Ben

3. All 10 coins start as heads. We want all tails. Each coin needs to flip an odd number of times to end up as tails.

Why the minimum is even:

Each move flips exactly 3 coins. After k moves, the total number of individual coin flips is $3k$.

For all 10 coins to show tails, each must be flipped an odd number of times. The sum of 10 odd numbers is always even.

So $3k$ must be even, meaning k must be even. The candidates are $k = 2, 4, 6, \ldots$

Can we do it in 2 moves?

Two moves give 6 flips total. To flip 10 coins each an odd number of times, the minimum sum is $10 \times 1 = 10$ flips. But we only have 6. Impossible.

Can we do it in 4 moves?

A working strategy: Flip $\{1, 2, 3\}$, then $\{1, 4, 5\}$, then $\{1, 6, 7\}$, then $\{8, 9, 10\}$.

Coin 1 is flipped 3 times (odd). Coins 2-7 are flipped once each (odd). Coins 8-10 are flipped once each (odd). All 10 coins show tails!

Answer: (B) 4

4. The claim is: "If vowel, then even number."

To disprove a conditional "If P then Q," you need to find a case where P is true but Q is false.

Card A: Has a vowel. Must check if the other side is even. If odd, the claim fails. **Must turn over.**

Card K: Not a vowel. The claim says nothing about consonants. Whatever is on the other side does not matter. **Do not turn over.**

Card 4: Even number. If there is a vowel on the other side, the claim is satisfied. If there is a consonant, the claim says nothing about it. Either way, no violation possible. **Do not turn over.**

Card 7: Odd number. If there is a vowel on the other side, the claim is violated (vowel with odd number). **Must turn over.**

This is the famous **Wason selection task**. Most people incorrectly choose A and 4, but the logical answer is A and 7.

Answer: (C) A and 7

5. This problem seems impossible at first. Adding just 10 meters to a 40,000 km rope? Surely the gap must be microscopic!

The surprising math:

Let R = Earth's radius. The tight rope has circumference $C = 2\pi R$.

The new rope has circumference $C + 10 = 2\pi(R + h)$, where h is the height above ground.

Solving: $2\pi R + 10 = 2\pi R + 2\pi h$

$10 = 2\pi h$

$h = \frac{10}{2\pi} \approx \frac{10}{6.28} \approx 1.59$ meters

The stunning insight: The answer does not depend on R at all! Whether the rope is around Earth, a basketball, or Jupiter, adding 10 meters of length always raises it about 1.6 meters.

This is counterintuitive because we expect Earth's huge size to matter. But circumference is linear in radius, so the extra length translates directly to extra radius.

Answer: (C) About 1.6 m

6. This is a beautiful **parity argument**.

Each handshake involves exactly two people. So if we add up "how many hands each person shook," each handshake gets counted twice.

Key insight: The sum of all the "handshake counts" must be even (since it equals twice the number of handshakes).

If all 7 people shook an odd number of hands, the sum of 7 odd numbers would be odd. But we just said this sum must be even!

Contradiction. So it is impossible for all 7 people to have shaken an odd number of hands.

General principle: In any graph, the number of vertices with odd degree must be even. With 7 vertices, you cannot have all 7 with odd degree.

Answer: (D) No, impossible

7. The number 142857 is the famous **cyclic number**. It is $\frac{1}{7}$ of 999999.

Calculate: $142857 \times 7 = ?$

Rather than multiply directly, notice:

$142857 = \frac{999999}{7}$

So $142857 \times 7 = 999999$.

Verification: $142857 \times 7 = 142857 \times (8 - 1) = 1142856 - 142857 = 999999$

Bonus fact: Multiplying 142857 by 1, 2, 3, 4, 5, or 6 gives a cyclic permutation of its digits:

- $142857 \times 1 = 142857$
- $142857 \times 2 = 285714$
- $142857 \times 3 = 428571$
- And so on...

But $\times 7$ gives all 9s!

Answer: (A) 999,999

Medium Questions (8–14)

8. The first player has a winning strategy, and it is beautifully simple.

The winning move: Player 1 places the first coin exactly at the center of the table.

The strategy: After that, whenever Player 2 places a coin anywhere, Player 1 places a coin at the point directly opposite the center (180° rotation).

Why this works:

The table has rotational symmetry about its center. If a position is valid for Player 2's coin, the opposite position is also valid for Player 1's response (by symmetry).

Player 1 can always respond. So Player 2 will eventually run out of valid positions first.

Key insight: By taking the center, Player 1 "owns" the symmetry. Every move Player 2 makes, Player 1 can mirror. Since Player 1 moved first and broke no symmetry, Player 2 runs out of moves first.

Answer: (A) First player wins

9. Your first instinct might be to minimize breaks by making clever cuts that split large pieces. But watch what actually happens.

The invariant insight:

Start: 1 piece (the whole bar).

After each break, the number of pieces increases by exactly 1.

End: 48 pieces (individual squares).

So you need exactly $48 - 1 = 47$ breaks.

No shortcuts exist.

It does not matter how you break: horizontally, vertically, or in any pattern. Each break adds exactly one piece. To go from 1 piece to 48 pieces requires exactly 47 breaks.

This is an **invariant** argument. The quantity "(number of breaks) + (number of pieces) = 48" is conserved throughout the process.

Answer: (D) 47

10. This is the **Frobenius coin problem** (also called the Chicken McNugget theorem).

For two coprime denominations a and b, the largest amount that cannot be made is $ab - a - b$.

With 5-cent and 7-cent stamps: $\gcd(5, 7) = 1$ (coprime).

Largest unmakeable: $5 \times 7 - 5 - 7 = 35 - 12 = 23$ cents.

Verification: 24 = 2(7) + 2(5) = 14 + 10. Correct. All amounts ≥ 24 can be made.

$23 = 5a + 7b$? Try: $a = 0, b = 23/7$ no; $a = 1, b = 18/7$ no; $a = 2, b = 13/7$ no; $a = 3, b = 8/7$ no; $a = 4, b = 3/7$ no. Cannot make 23!

Answer: (B) 23 cents

11. Let $x = \sqrt{6 + \sqrt{6 + \sqrt{6 + \cdots}}}$.

The key insight: since the pattern repeats infinitely, the value inside the outer square root is also x.

So $x = \sqrt{6 + x}$.

Square both sides: $x^2 = 6 + x$.

Rearrange: $x^2 - x - 6 = 0$.

Factor: $(x - 3)(x + 2) = 0$.

Solutions: $x = 3$ or $x = -2$.

Since x is a square root (hence non-negative), $x = 3$.

Answer: (C) 3

12. The square has all $90°$ angles. The equilateral triangle has all $60°$ angles.

At the vertex where the triangle and square meet (not on the shared side), we have:

- One angle from the square: $90°$
- One angle from the triangle: $60°$

Wait, the question asks about the angle between a non-shared side of the triangle and an adjacent side of the square.

At the corner where they meet: the interior angle of the square ($90°$) and the interior angle of the triangle ($60°$) are both on the same side of the shared edge.

The angle between the non-shared edges is the **exterior** angle, which equals:

$360° - 90° - 60° = 210°$... but that is the reflex angle.

The non-reflex angle between these sides is $90° + 60° = 150°$.

Think of it this way: The two shapes sit on opposite sides of the shared edge. Their interior angles add up on one side, so the angle between the outer edges (measured on the outside) is $90° + 60° = 150°$.

Answer: (D) $150°$

13. This is the classic **mutilated chessboard** problem.

First instinct: Try to tile it. You might find patterns that almost work but always leave gaps.

The key insight: color!

A standard chessboard has 32 white squares and 32 black squares.

Diagonally opposite corners are the SAME color. (Check: a1 and h8 are both one color.)

Removing two same-colored squares leaves 30 of one color and 32 of the other.

Why domino tiling is impossible:

Each domino covers exactly one white square and one black square. So 31 dominoes would cover 31 white and 31 black squares.

But we have 30 of one color and 32 of the other!

No matter how you try, you cannot cover 30 squares of one color and 32 of another using dominoes that cover one of each.

This is a **parity/coloring** argument: sometimes what seems like a geometry problem is really about an invariant.

Answer: (C) No, parity prevents it

14. Your intuition might say: "+10% and −10% should cancel out to zero change."

Let us check with actual numbers.

Start with \$100.

After $+10\%$**:** $100 \times 1.10 = \$110$

After -10%**:** $110 \times 0.90 = \$99$

We lost \$1! The investment is worth less.

Why does this happen?

The gain is 10% of the original (\$100), adding \$10.

The loss is 10% of the increased amount (\$110), subtracting \$11.

You lose more than you gain because percentages compound asymmetrically.

The math: $1.10 \times 0.90 = 0.99$, not 1.00.

In general, $(1 + x)(1 - x) = 1 - x^2 < 1$ for any $x \neq 0$.

Order does not matter: Whether you gain then lose, or lose then gain, the result is the same: $0.99 \times$ original.

Answer: (B) Worth less

Hard Questions (15–21)

15. Understanding $10^{100} - 47$**:**

10^{100} is a 1 followed by 100 zeros: $1\underbrace{00\ldots0}_{\text{100 zeros}}$.

When we subtract 47, we are essentially subtracting from the last few digits.

How subtraction works with large powers of 10:

Think of 10^{100} as $\underbrace{99\ldots9}_{\text{100 nines}} + 1$.

So $10^{100} - 47 = \underbrace{99\ldots9}_{\text{100 nines}} + 1 - 47 = \underbrace{99\ldots9}_{\text{100 nines}} - 46$.

When we subtract 46 from a string of nines, we get:

$\underbrace{99...9}_{\text{98 nines}}53$ (the last two digits become $99 - 46 = 53$, and we do not need to borrow further).

Counting the digits:

The result has 98 nines followed by 53.

Total digits: $98 + 2 = 100$ digits.

Computing the digit sum:

$98 \times 9 + 5 + 3 = 882 + 8 = 890$.

Verification: $10^{100} - 47$ should have digit sum $\equiv 10^{100} - 47 \pmod 9$. Since $10 \equiv 1 \pmod 9$, $10^{100} \equiv 1 \pmod 9$. So $10^{100} - 47 \equiv 1 - 47 \equiv -46 \equiv 8 \pmod 9$. Check: $890 = 98 \times 9 + 8 \equiv 8 \pmod 9$. Correct!

Answer: (B) 890

16. The key insight:

This is the famous "bird between trains" puzzle. The bird flies back and forth between the trains until they meet.

You might think you need to track each leg of the bird's journey. But there is a much simpler approach!

How long until the trains meet?

The trains approach each other at a combined speed of $50 + 100 = 150$ km/h.

Starting 500 km apart:

Time to meet $= \frac{500}{150} = \frac{10}{3}$ hours $= 3$ hours 20 minutes.

How far does the bird fly?

The bird flies continuously at 150 km/h for the entire time until the trains meet.

Distance $=$ speed $\times$ time $= 150 \times \frac{10}{3} = 500$ km.

The elegant solution:

We did not need to track the bird's zigzag path! It does not matter when or how often the bird turns around. The bird flies for $\frac{10}{3}$ hours at 150 km/h, period.

Answer: (C) 500 km

17. The classic census taker puzzle:

This is one of the most famous logic puzzles. Let us work through it step by step.

Step 1: Find all factorizations of 36 into three positive integers.

Ages	Sum
$1, 1, 36$	38
$1, 2, 18$	21
$1, 3, 12$	16
$1, 4, 9$	14
$1, 6, 6$	13
$2, 2, 9$	13
$2, 3, 6$	11
$3, 3, 4$	10

Step 2: Why does the census taker need more information?

The census taker knows the house number (the sum). If the sum uniquely determined the ages, the puzzle would be solved.

But the census taker still does not know! This means the sum is ambiguous.

Only one sum appears twice: **13** (for ages 1,6,6 and 2,2,9).

Step 3: Use the final clue.

"My oldest plays piano" tells us there IS an oldest child (a unique maximum age).

$(1, 6, 6)$: Two children are 6, so no unique "oldest."

$(2, 2, 9)$: The 9-year-old is uniquely oldest.

The ages are 2, 2, and 9.

Answer: (B) 2, 2, 9

18. The clever twist: weights on BOTH sides

Unlike a typical weighing problem where you only put weights on one side, here you can put weights on either pan. This dramatically increases what you can measure.

Why powers of 3?

With weights 1, 3, 9, 27 (powers of 3), each weight has three options:

- On the weights' side (adds to balance the object)
- Off the scale (contributes nothing)
- On the object's side (effectively subtracts)

This is called **balanced ternary** representation.

Example: weighing 5 grams

Put the 9 on the weights' side and the 1, 3 on the object's side.

Balance equation: object + 1 + 3 = 9, so object = 5 grams.

Example: weighing 2 grams

Put the 3 on the weights' side and the 1 on the object's side.

Balance equation: object + 1 = 3, so object = 2 grams.

Finding the maximum:

To weigh the heaviest possible object, put ALL weights on the weights' side.

Maximum = $1 + 3 + 9 + 27 = 40$ grams.

Bonus insight: With n powers of 3, you can weigh any integer from 1 to $\frac{3^n-1}{2}$. For $n = 4$: $\frac{81-1}{2} = 40$.

Answer: (D) 40

19. This is a **derangement** problem: we want arrangements where NO student draws their own name.

Building up the pattern:

Let D_n = number of derangements of n items.

$D_1 = 0$**:** With 1 person, there is no way to move them to a different seat.

$D_2 = 1$**:** With 2 people (A, B), swap them: B sits in A's seat, A sits in B's seat.

$D_3 = 2$**:** With 3 people (A, B, C), the derangements are: BCA and CAB.

The recursion:

Here is a clever pattern: $D_n = (n-1)(D_{n-1} + D_{n-2})$.

Why? Person 1 can go to any of $(n-1)$ other seats. Say person 1 goes to seat 2. Now, person 2 either:

- Goes to seat 1 (they swap), leaving $n-2$ people to derange: D_{n-2} ways.
- Goes to some other seat, which is like deranging $n-1$ people: D_{n-1} ways.

Computing:

$D_3 = 2 \times (D_2 + D_1) = 2 \times (1 + 0) = 2$

$D_4 = 3 \times (D_3 + D_2) = 3 \times (2 + 1) = 9$

$D_5 = 4 \times (D_4 + D_3) = 4 \times (9 + 2) = 44$

Answer: (C) 44

20. This problem tests **exponential thinking**, and most people get it wrong!

The trap: Your intuition might say "one-eighth is early, maybe around day 3 or 4."

The insight: Work backwards from day 30.

Day 30: covers the whole pond (1)

Day 29: covers half the pond ($\frac{1}{2}$)

Day 28: covers one-quarter of the pond ($\frac{1}{4}$)

Day 27: covers one-eighth of the pond ($\frac{1}{8}$)

Why this works:

If the lily pad doubles each day, then yesterday it was half the size. Going back 3 days from "full" means going through $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{8}$.

The lesson:

Exponential growth is counterintuitive. Most of the growth happens at the very end. On day 27, the pond looks almost empty (only 12.5% covered). Just 3 days later, it is completely full!

Answer: (D) Day 27

21. This puzzle seems like it should depend on spoon size, stirring, or the order of transfers. Surprisingly, it does not!

The elegant insight:

Both jars end with 100 ml total (we just swapped some liquid).

Let us say jar A (originally all milk) ends with x ml of water in it.

Then jar A has $(100 - x)$ ml of milk remaining.

The "missing" milk from A went to jar B. So jar B has x ml of milk.

Conclusion:

Milk in B = x ml

Water in A = x ml

They are exactly equal!

Why this works regardless of spoon size:

Whatever amount of milk "emigrates" to jar B, an equal volume of water must "immigrate" to jar A (since both jars maintain 100 ml total).

Verification with numbers:

Say the spoon holds 10 ml.

Transfer 1: Move 10 ml milk from A to B. Now A has 90 ml milk; B has 100 ml water + 10 ml milk.

Transfer 2: B is $\frac{10}{110}$ milk. A spoonful contains $\frac{10}{110} \times 10 = \frac{100}{110} \approx 0.91$ ml milk and $\frac{100}{110} \times 10 \approx 9.09$ ml water.

After transfer 2: A has $90 + 0.91 = 90.91$ ml milk and 9.09 ml water.

B has $10 - 0.91 = 9.09$ ml milk.

Water in A = milk in B = 9.09 ml. Equal!

Answer: (C) They are equal

End of Free Sample

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